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NONLINEAR ANALYSIS AND MODELING OF TIRES

Submitted to:

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ABSTRACT

The objective of the study was to develop efficient modeling techniques and computational strategies for: a) predicting the nonlinear response of tires subjected to inflation pressure, mechanical and thermal loads; b) determining the footprint region, and analyzing the tire pavement contact problem, including the effect of friction; and c) determining the sensitivity of the tire response (displacements, stresses, strain energy, contact pressures and contact area) to variations in the different material and geometric parameters. Two computational strategies were developed. In the first strategy the tire was modeled by using either a two-dimensional shear flexible mixed shell finite elements or a quasi-three-dimensional solid model. The contact conditions were incorporated into the formulation by using a perturbed Lagrangian approach. A number of model reduction techniques were applied to substantially reduce the number of degrees of freedom used in describing the response outside the contact region.

The second strategy exploited the axial symmetry of the undeformed tire, and uses cylindrical coordinates in the development of three-dimensional elements for modeling each of the different parts of the tire cross section. Model reduction techniques are also used with this strategy.

The principal investigator was Dr. Ahmed K. Noor, Ferman W. Perry Professor of Aerospace Structures and Applied Mechanics, and the NASA Technical Officer was Dr. John A. Tanner, Impact Dynamics Branch, Structural Dynamics Division, NASA Langley Research Center.

RESEARCH AND OTHER ACCOMPLISHMENTS

The research activities conducted under this grant are briefly described subsequently. Seven publications were completed. Four technical presentations were made based on the studies made under this grant. The list of publications and presentations are given after this summary. The abstracts of the papers are given in the Appendix, followed by a copy of each paper.

In addition, a workshop was organized in October 1994 to assess the state-of-the-technology in tire modeling and to identify mechanisms for rapid transfer of the research results to industry. The participants of the workshop came from NASA, tire industry, government agencies and universities. The proceedings of the workshop were published in August 1995 (Ref. 5).

The research activities can be grouped into three major tasks: a) accurate determination of the characteristics of the tire cross section using scanned images; b) development of model-size reduction strategies for the efficient simulation of the tire contact response, and its sensitivity with respect to variations in the material and geometric parameters of the tire constituents (cord, rubber and bead); and c) development of three-dimensional elements in cylindrical coordinates for modeling the tire.

In the first task, the tire cross section image is captured using a flat bed scanner. The raw image is digitally enhanced by subtracting a smooth area-averaged copy from the original. The image consists of a grid of pixels, each with its own gray scale value (amount of black and white). The gray scale values are used to determine geometrical details, since geometry changes correspond to changes in color. An edge tracing procedure is used to determine the inner and outer surfaces. Also, the tire cord locations, and ends per inch (epi), are determined by examining the changes in the color of the image. The details of this procedure being prepared for publication.

An interactive computer program has been developed which allows the user to generate, in a semi-automated manner, the geometrical data of the tire cross section for use in the finite element model.

The second task included development of a number of model-reduction strategies, including:

- 1) reduced basis technique for simulating the tire contact response. This strategy is an extension of the reduced basis technique originally developed by the principal

investigator for the solution of geometrically nonlinear problems. It is based on significantly reducing the number of degrees of freedom away from the contact zone.

2) partitioning strategy based on dividing the tire into two regions: the contact region, and the region away from the contact area. A preconditioned conjugate gradient approach is used with the preconditioning matrix selected to correspond to the degrees of freedom in the contact region.

3) neglecting the changes in the tire response away from the contact area.

The first two strategies have been implemented into an in-house computer program.

The third task included the development of three-dimensional finite elements in cylindrical coordinates for modeling tires. In contrast to elements formulated in Cartesian coordinates, these elements allow the exact representation of circular shapes. Numerical results are presented for the application of this formulation to the analysis of a radial passenger tire subjected to rim mounting, inflation pressure, and rigid pavement contact. Comparison was made with both available experimental data, and the predictions of a commercial code using finite elements developed in Cartesian coordinates.

PRESENTATIONS

1. Noor, A. K., Tanner, J. A. and Peters, J. M., "Sensitivity of Tire Response to Variations in Material and Geometric Parameters," International Conference on Computational Engineering Science (ICES'91), Patras, Greece, April 21-25, 1991.
2. Noor, A. K., Tanner, J. A. and Peters, J. M., "Reduced Basis Technique for Evaluating the Sensitivity Coefficients of the Nonlinear Tire Response," Tire Society 11th Annual Meeting and Conference on Tire Science and Technology, University of Akron, Akron, OH, March 24-25, 1992.
3. Noor, A. K., "Advances in Reduction Techniques for Tire Contact Problems," Workshop on Computational Modeling of Tires, Hampton, VA, Oct. 26-27, 1994.
4. Danielson, K. T. and Noor, A. K., "Finite Elements Developed in Cylindrical Coordinates for Three-Dimensional Tire Analysis," Tire Society 15th Annual Meeting and Conference on Tire Science and Technology, University of Akron, Akron, OH, March 19-20, 1996.

REFERENCES

1. Noor, A. K., Tanner, J. A. and Peters, J. M., "Sensitivity of Tire Response to Variations in Material and Geometric Parameters," *Finite Elements in Analysis and Design*, Vol. 11, 1992, pp. 77-86.
2. Noor, A. K., Tanner, J. A., Peters, J. M. and Robinson, M. P., "Analytical Studies of the Space Shuttle Orbiter Nose-Gear Tire," SAE Technical Paper Series, 1991 SAE Aerospace Atlantic, Dayton, OH, April 22-26, 1991, Paper No. 91198.
3. Noor, A. K., Tanner, J. A. and Peters, J. M., "Reduced Basis Technique for Evaluating the Sensitivity Coefficients of the Nonlinear Tire Response," *AIAA Journal*, Vol. 31, No. 2, Feb. 1993, pp. 370-376.
4. Noor, A. K. and Peters, J. M., "Reduction Technique for Tire Contact Problems," Workshop on Computational Tire Modeling, Hampton, VA, Oct. 26-27, 1994, NASA CP-3306, 1994, pp. 69-88; *Computers and Structures* (to appear).
5. Noor, A. K. and Tanner, J. A. (compilers), *Computational Modeling of Tires*, NASA CP-3306, August 1995.
6. Danielson, K. T., Noor, A. K. and Green, J. S., "Computational Strategies for Tire Modeling and Analysis," *Computers and Structures* (to appear).
7. Danielson, K. T. and Noor, A. K., "Finite Elements Developed in Cylindrical Coordinates for Three-Dimensional Tire Analysis," *Tire Science and Technology* (to appear).

Appendix
ABSTRACTS OF PUBLICATIONS

**Sensitivity of Tire Response to Variations in Material
and Geometric Parameters**

Ahmed K. Noor, John A. Tanner and Jeanne M. Peters

A computational procedure is presented for evaluating the analytic sensitivity derivatives of the tire response with respect to material and geometric parameters of the tire. The tire is modeled by using a two-dimensional laminated anisotropic shell theory with the effects of variation in material and geometric parameters included. The computational procedure is applied to the case of uniform inflation pressure on the space shuttle nose-gear tire when subjected to uniform inflation pressure. Numerical results are presented showing the sensitivity of the different response quantities to variations in the material characteristics of both the cord and the rubber.

Finite Elements in Analysis and Design, Vol. 11, 1992, pp. 77-86.

Analytical Studies of the Space Shuttle Orbiter Nose-Gear Tire

Ahmed K. Noor, John A. Tanner, Jeanne M. Peters and Martha P. Robinson

A computational procedure is presented for evaluating the analytic sensitivity derivatives of the tire response with respect to material and geometrical properties of the tire. The tire is modeled by using a two-dimensional laminated anisotropic shell theory with the effects of variation in material and geometric parameters included. The computational procedure is applied to the case of the Space Shuttle orbiter nose-gear tire subjected to uniform inflation pressure. Numerical results are presented which show the sensitivity of the different tire response quantities to variations in the material characteristics of both the cord and rubber.

SAE Technical Paper Series, 1991 SAE Aerospace Atlantic, Dayton, OH, April 22-26, 1991, Paper No. 91198.

**Reduced Basis Technique for Evaluating the Sensitivity Coefficients
of the Nonlinear Tire Response**

Ahmed K. Noor, John A. Tanner and Jeanne M. Peters

An efficient reduced basis technique is presented for calculating the sensitivity of nonlinear tire response to variations in the design variables. The tire is discretized by using three-field mixed finite element models. The vector of structural response and its first-order and second-order sensitivity coefficients (derivatives with respect to design variables) are each expressed as linear combinations of a small number of basis (or global approximation) vectors. The Bubnov-Galerkin technique is then used to approximate each of the finite element equations governing the response, and the sensitivity coefficients, by a small number of algebraic equations in the amplitudes of these vectors. Path derivatives (derivatives of the response vector with respect to path parameters, e.g., load parameters) are used as basis vectors for approximating the response. A combination of the path derivatives and their derivatives with respect to the design variables is used for approximating the sensitivity coefficients. The potential of the proposed technique is discussed and its effectiveness is demonstrated by means of a numerical example of the Space Shuttle nose-gear tire subjected to uniform inflation pressure. The design variables are selected to be the material properties of the cord and rubber as well as the cord diameters, end counts, and angles.

AIAA Journal, Vol. 31, No. 2, Feb. 1993, pp. 370-376.

Computational Modeling of Tires

Ahmed K. Noor and John A. Tanner (compilers)

A significant capability has been developed over the past two decades for the computational modeling and analysis of tires. Several in-house and commercial software systems are currently available for accurate determination of static load deflection characteristics, contact area shape, free vibration modes, and quasi-static rolling resistance. This success can be attributed, in part, to the joint NASA/U.S. tire industry National Tire Modeling Program (NTMP). This program was initiated in 1983 and was completed in 1991. At its peak seven U.S. tire companies, four universities, and a software developer participated in the program. The NTMP established a set of benchmark tire modeling problems, developed a national database of experimental tire measurements, and led to the development of a number of tire modeling strategies and solution algorithms that could be incorporated into new or existing software packages.

Despite the significant advances made in the computational modeling and analysis of tires, a number of technology needs and related tasks must be addressed by the research community to enhance the state-of-the-art in computational modeling of tires and to make it an effective tool in tire design. The joint University of Virginia/NASA workshop held at the Peninsula Graduate Engineering Center of the Virginia Consortium of Engineering and Science Universities, Oct. 26-27, 1994 focused on the status of computational modeling of tires and the current and future needs for further development of this technology. The list of pacing items given in this introduction was compiled from a number of participants.

The technology needs identified by the participants can be grouped into the following five major headings: 1) material characterization of filled rubber and heat generation mechanisms; 2) understanding the physical phenomena associated with frictional contact, wear and noise generation; 3) accurate determination of tire footprint shape, size and pressure distribution; 4) prediction of wear resistance, failure and fatigue; and 5) tire/vehicle interaction and its effect on riding and handling qualities and performance.

For each of the aforementioned items, attempts should be made to exploit the major characteristics of high-performance computing technologies, as well as the future computing environment. The five primary technology needs and related tasks are described subsequently.

NASA CP-3306 - Proceedings of the workshop held at NASA Langley Research Center, Hampton, VA, Oct. 26-27, 1994.

Reduction Technique for Tire Contact Problems

Ahmed K. Noor and Jeanne M. Peters

A reduction technique and a computational procedure are presented for predicting the tire contact response and evaluating the sensitivity coefficients of the different response quantities. The sensitivity coefficients measure the sensitivity of the contact response to variations in the geometric and material parameters of the tire. The tire is modeled using a two-dimensional laminated anisotropic shell theory with the effects of variation in geometric and material parameters, transverse shear deformation, and geometric nonlinearities included. The contact conditions are incorporated into the formulation by using a perturbed Lagrangian approach with the fundamental unknowns consisting of the stress resultants, the generalized displacements, and the Lagrange multipliers associated with the contact conditions. The elemental arrays are obtained by using a modified two-field, mixed variational principle.

For the application of the reduction technique, the tire finite element model is partitioned into two regions. The first region consists of the nodes that are likely to come in contact with the pavement, and the second region includes all the remaining nodes. The

reduction technique is used to significantly reduce the degrees of freedom in the second region.

The effectiveness of the computational procedure is demonstrated by a numerical example of the frictionless contact response of the space shuttle nose-gear tire, inflated and pressed against a rigid flat surface.

NASA CP-3306, pp. 69-88.

Computational Strategies for Tire Modeling and Analysis

Kent T. Danielson, Ahmed K. Noor and James S. Green

Computational strategies are presented for the modeling and analysis of tires in contact with pavement. A procedure is introduced for simple and accurate determination of tire cross-sectional geometric characteristics from a digitally scanned image. Three new strategies for reducing the computational effort in the finite element solution of tire-pavement contact are also presented. These strategies take advantage of the observation that footprint loads do not usually stimulate a significant tire response away from the pavement contact region. The finite element strategies differ in their level of approximation and required amount of computer resources. The effectiveness of the strategies is demonstrated by numerical examples of frictionless and frictional contact of the space shuttle Orbiter nose-gear tire. Both an in-house research code and a commercial finite element code are used in the numerical studies.

Computers and Structures (to appear).

Finite Elements Developed in Cylindrical Coordinates for Three-Dimensional Tire Analysis

Kent T. Danielson and Ahmed K. Noor

Finite elements developed in cylindrical coordinates are presented for three-dimensional analysis of tires. In contrast to elements formulated in Cartesian coordinates, these elements allow the exact representation of circular shapes. The exact modeling of circular geometries can provide better finite element predictions and reduce the number of elements needed around the tire circumference. Numerical results are presented for the application of this formulation to the analysis of a radial passenger tire subjected to rim mounting, nonconservative inflation pressure, and rigid pavement contact. The predictions of the foregoing finite elements are compared to experimental data and to predictions of a commercial code using finite elements developed in Cartesian coordinates. The comparisons demonstrate the accuracy and the advantages of the cylindrical coordinate formulation for three-dimensional finite element analysis of tires.

Tire Science and Technology (to appear).

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